



first-principles electronic structure  
calculations for the solid state

some remarks

# first-principles calculations

most of the techniques used for molecules can also be applied to the solid state: DFT, HF, MBPT, CC

e.g., DFT

$$E[n] = T[n] + \int \sum_l \frac{Z_l n(\mathbf{r})}{|\mathbf{r} - \mathbf{R}_l|} d\mathbf{r} + \frac{1}{2} \iint \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} d\mathbf{r} d\mathbf{r}' + E_{xc}[n]$$

charge density  $n(\mathbf{r}) = \sum_{n,\mathbf{k}} |\psi_{n,\mathbf{k}}(\mathbf{r})|^2$

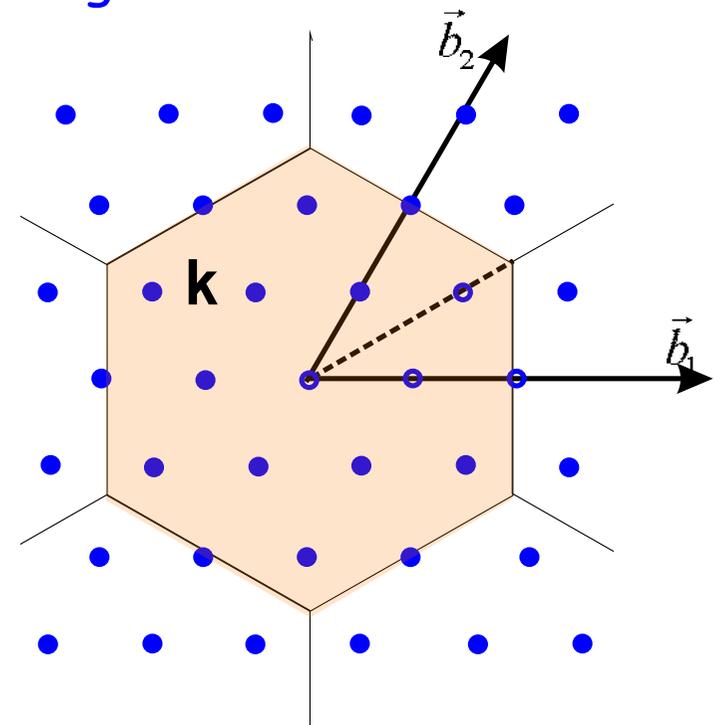
needs a Brillouin zone integration

➤ metals need a dense  $\mathbf{k}$ -point grid

self-consistency:

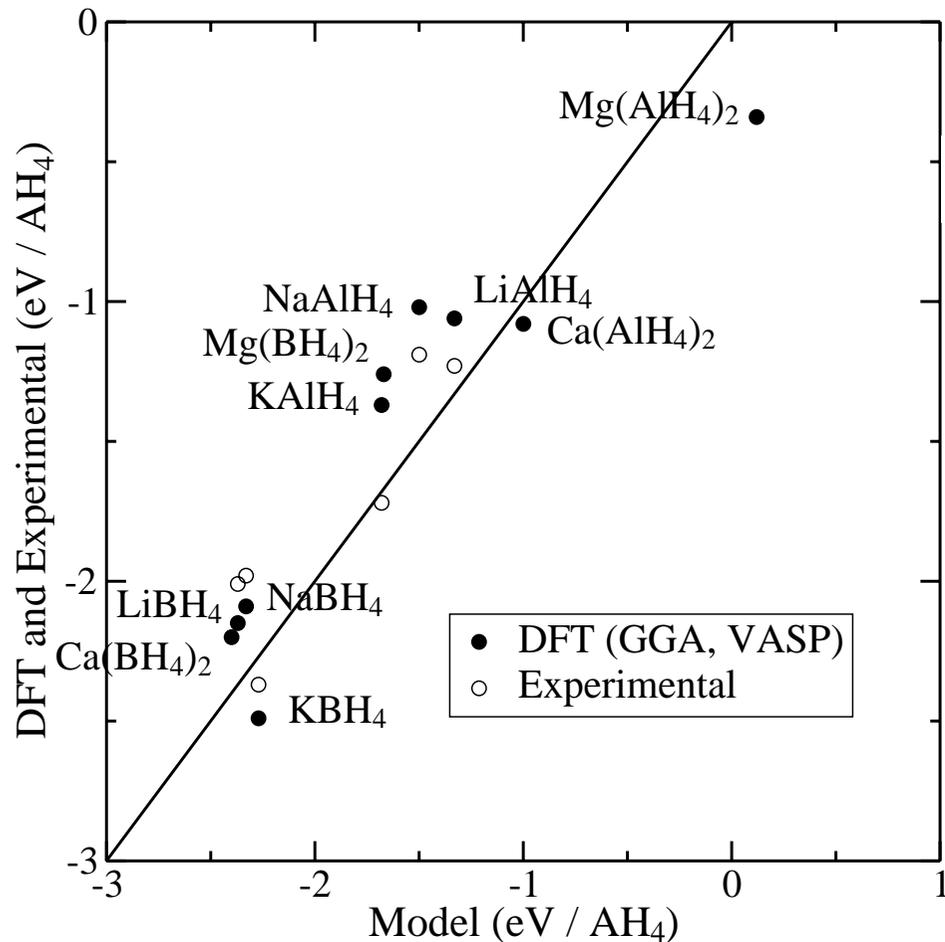
➤ metals converge slowly

➤ magnetic metals even more so



# the work horse: Density Functional Theory (DFT)

formation energies / heats of formation covalent/ionic bonding  
calculated with DFT/GGA (PBE) are typically accurate  
on a scale of 0.1 eV (10 kJ/mol)



... but the error can be larger ...

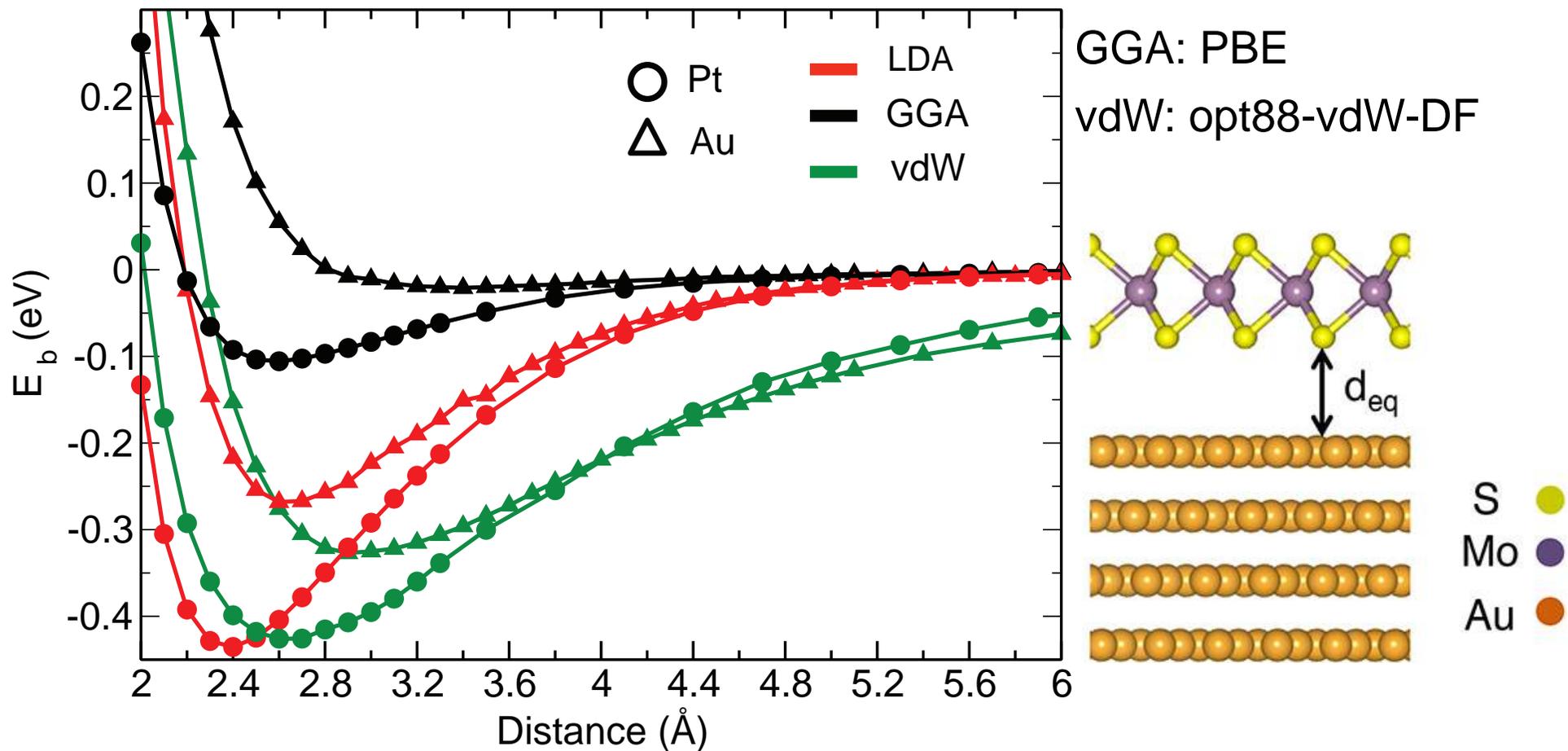
formation energy MgH<sub>2</sub>

0.57 eV/H<sub>2</sub> (GGA/PBE/ZPE)

0.76 eV/H<sub>2</sub> (Exp)

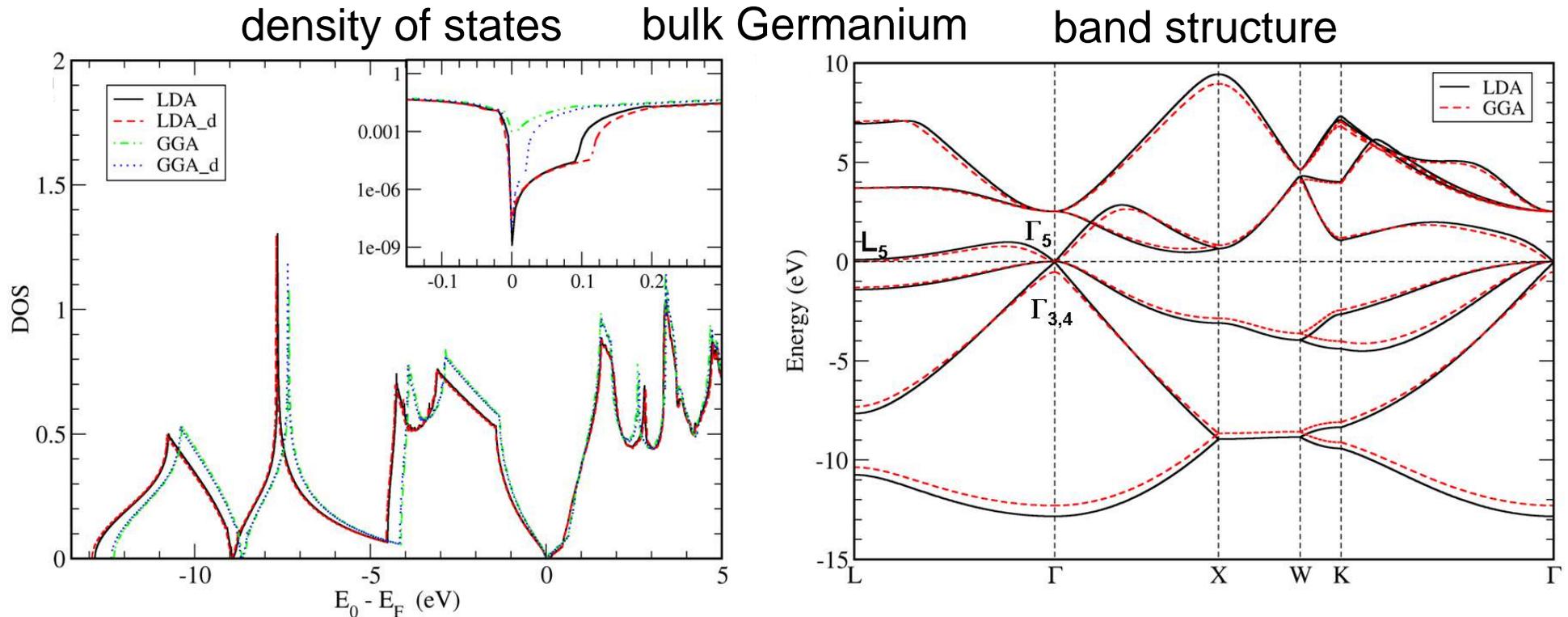
# Density Functional Theory (DFT)

weak interactions need a van der Waals functional



# Density Functional Theory (DFT)

spectrum obtained from DFT/GGA (LDA) is not so impressive



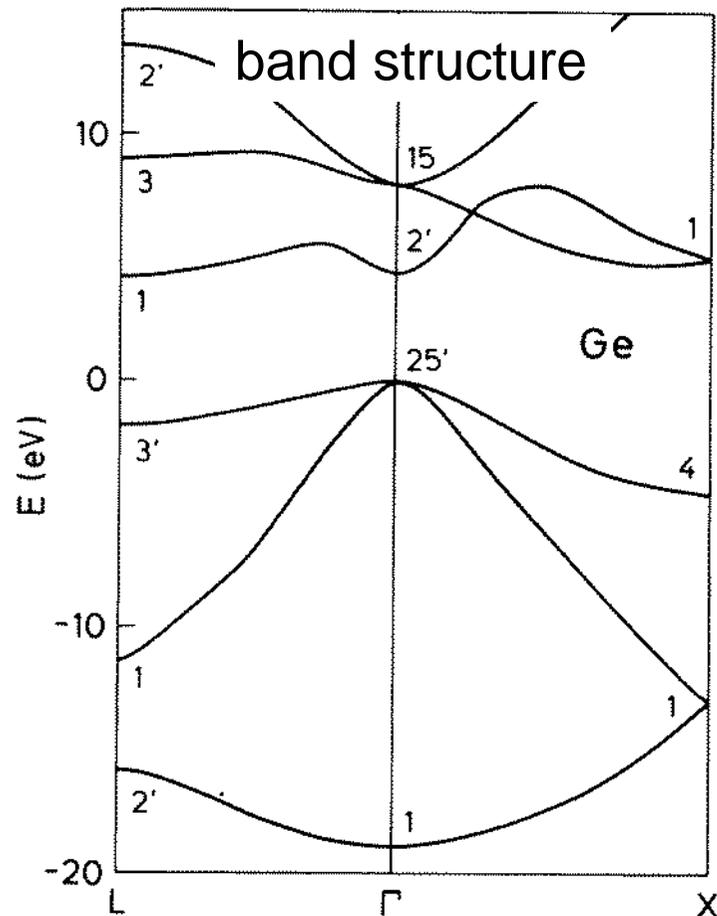
exp.:      Ge is a **semiconductor** with band gap 0.7 eV

GGA/LDA:      Ge is a **metal**

band dispersions are quite reasonable  $\sim 1-10\%$

# Hartree-Fock (HF)

HF computationally more expensive than DFT: 1-2 orders of magnitude



HF total energies are worse than DFT

the spectrum obtained from HF  
is even less impressive

exp: Ge semiconductor with band gap 0.7 eV

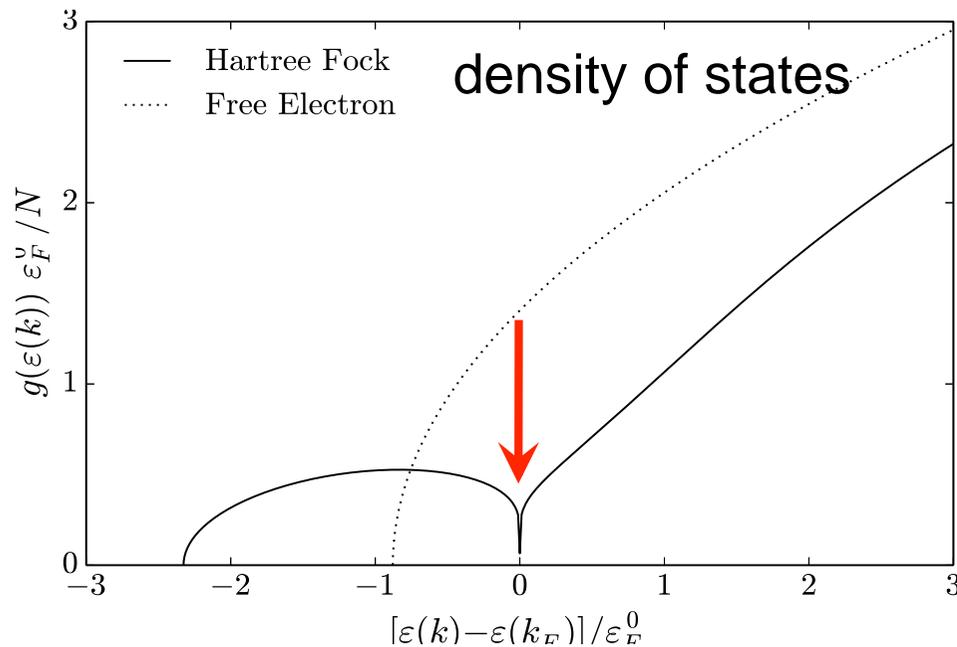
HF: Ge insulator with band gap 4.2 eV

band dispersions unreasonable ~50% too wide

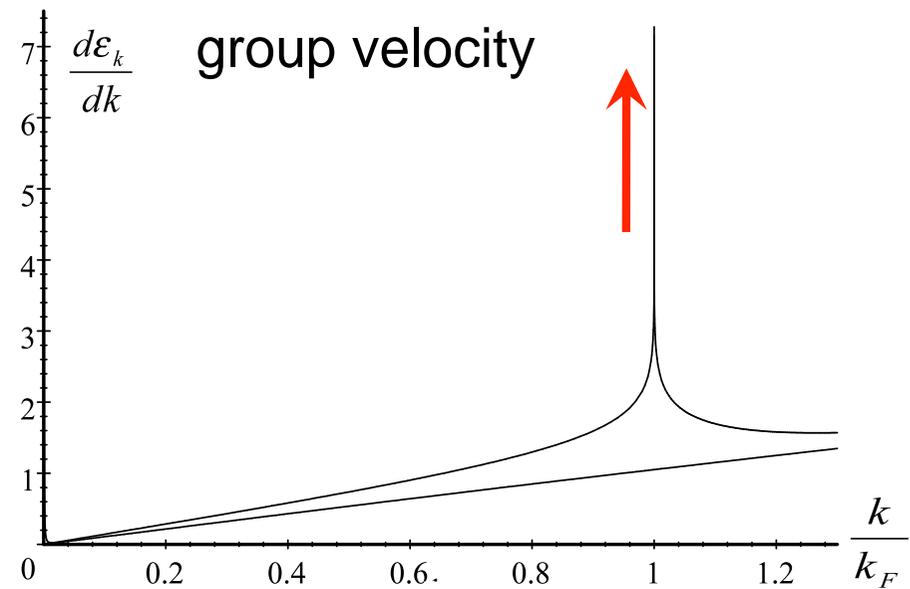
# Hartree-Fock (HF): from bad to worse

For metals the spectrum obtained from HF is a disaster

e.g. homogeneous electron gas (jellium) free electron spectrum  $\approx$  MBPT



density of states  
at  $E_F$  is zero



electron velocity  
at  $E_F$  is infinity

metals: electrons with energy  $E \approx E_F$  are doing the conduction

# Electron correlations

truncated correlated methods don't work well for extended systems

$$\text{MP2} = \text{[diagram: square with diagonal arrows]} + \text{[diagram: two stacked circles]} \quad \text{for metals} \rightarrow \infty$$

$$\text{CI SDT...} \rightarrow 0$$

infinite summation methods work

$$\text{RPA} = \text{[diagram: two stacked circles]} + \text{[diagram: three stacked circles]} + \text{[diagram: four stacked circles]} + \dots$$

coupled cluster      full CI

all these methods are computationally very expensive